Secular Evolution in Barred Galaxies

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Abstract. A strong bar rotating within a massive halo should lose angular momentum to the halo through dynamical friction, as predicted by Weinberg. We have conducted fully self-consistent, numerical simulations of barred galaxy models with a live halo population and find that bars are indeed braked very rapidly. Specifically, we find that the bar slows sufficiently within a few rotation periods that the distance from the centre to co-rotation is more than twice the semi-major axis of the bar. Observational evidence (meagre) for bar pattern speeds seems to suggest that this ratio typically lies between 1.2 to 1.5 in real galaxies. We consider, a number of possible explanations for this discrepancy between theoretical prediction and observation, and conclude that no conventional alternative seems able to account for it.

1 Introduction

Chandrasekhar (1943) showed that a massive object moving though a background "sea" of light particles would experience a drag. The force is the gravitational attraction by the wake produced by the motion of the massive object (see e.g., Binney & Tremaine 1987, §7.1). When the mass, M, of the perturber is much larger than that of the individual background particles, the acceleration takes the form

$$\frac{dv_M}{dt} \propto -\rho\, M\, f\left(\frac{v_M}{\sigma}\right), \tag{1}$$

where ρ is the background density and f is a function of the ratio of the perturber's velocity, v_M, to the (assumed isotropic) velocity dispersion, σ, of the background particles. For a Maxwellian distribution of velocities, this function is a maximum when $v_M \simeq 1.37\sigma$.

A similar process must occur as a massive bar rotates inside a halo. In this case, the bar creates a wake in the halo which lags the bar and the gravitational attraction between the bar and the wake produces a torque which removes angular momentum from the bar and adds it to

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the halo. Weinberg (1985), adapting the perturbation theory approach of Tremaine & Weinberg (1984a), estimated the magnitude of the frictional drag force. Assuming a massive, rigid bar rotating in an isothermal halo, he concluded that the spin-down time for the bar would be as little as five bar rotations, for reasonable parameters!

Early low-quality, but fully self-consistent disc-halo simulations (Sellwood 1980) had previously revealed a rapid loss of angular momentum to the halo once a bar formed in the disc, at a rate roughly consistent with Weinberg's prediction. To our knowledge, no other such simulations have been conducted in the past 15 years; Combes et al. (1990) had only a bulge, not an extensive halo, of live particles, Raha et al. (1991) did not evolve their models for long enough, the "halo" particles of Little & Carlberg (1991) were confined to a plane, and the bar used by Hernquist & Weinberg (1992) was rigid and their model had no disc. We here report new fully self-consistent, simulations superior in many respects to those of Sellwood (1980), which were designed to reproduce the expected dynamical friction and to determine the secular changes to the bar, disc and halo in the long term. Athanassoula (work in progress) is conducting similar experiments using a direct N-body code on a GRAPE device.

2 New Simulations

2.1 Initial Set-up and Numerical Details

We begin by setting up a disc-halo equilibrium. The disc particles represent 30% of the total mass and are laid down with the Kuz'min-Toomre surface density distribution

$$\Sigma(R) = \frac{Mq}{2\pi a^2}\left(1 + \frac{R^2}{a^2}\right)^{-3/2}. \tag{2}$$

We truncate this profile sharply at the rather small radius of $R = 4a$ in order to limit particle loss from the grid at later times. We also disperse the disc particles about the mid-plane in a Gaussian fashion having a uniform rms thickness of $0.4a$.

The remaining 70% of the mass is represented by the "halo" particles, a dynamically uniform population which could be thought of as comprising both a luminous bulge and a more extended dark halo. The halo particles are set in equilibrium in the manner first used by Raha et al. (1991). They are selected from an isotropic DF having a (lowered) polytropic form $f = f[(-E)^m]$, with the limiting energy $E = \Phi_m$, being the potential at some limiting radius. The combined disc and halo gravitational potential

Fig. 1. The circular velocity curve at the start of the simulation (solid curve). The separate contributions from the disc (dot-dashed curve) and bulge + halo (dashed curve) are also shown.

distribution to be used in this DF is determined iteratively in the manner adopted by Prendergast & Tomer (1970) and Jarvis & Freeman (1985). The polytropic index $m = 1.5$ in our case; n.b. this corresponds to a standard $n = 3$ polytrope, where $m = n - \frac{3}{2}$ (Binney & Tremaine 1987). The resulting halo mass distribution is not far from spherical and, when combined with the disc, gives rise to the circular velocity curve in the mid-plane shown in Figure 1.

Having determined the potential of our initial mass distribution, we set the disc particles in motion. Their initial orbits are almost circular, but have enough random motion to maintain the vertical thickness and to set Toomre's $Q = 0.1$, in the case we focus on here. We have run other models in which $Q \geq 1$ at the start and find that the essential results we describe here are independent of the initial Q value.

Our simulations are performed on a 3-D Cartesian grid having 129^3 cubic cells; the code used was described by Sellwood & Merritt (1994). We set the length scale $a = 5$ mesh spaces, and chose a time step for

the leap-frog integration of 0.05 times the dynamical time $\sqrt{a^3/GM}$. We employ 300K equal mass particles, of which 90K represent the disc. We adopt units such that $G = M = a = 1$; the rotation period of a particle at the disc half-mass radius ($R \sim 2.3$) is about 35 in these units.

2.2 Evolution

This model is deliberately designed to be unstable and forms a strong, rapidly rotating bar within the first 100 dynamical times. As soon as the bar forms, a strong torque develops that begins to reduce the total angular momentum of the disc and to set the halo into rotation, as shown in Figure 2(a). Total angular momentum is conserved, of course; almost all that which is lost from the disc goes into the halo, the tiny remainder being carried away by escaping particles.

A bi-symmetric distortion is readily detectable in the distribution of halo particles which initially lags the bar by $\sim 45°$. As the evolution proceeds, both the torque and the lag angle gradually decrease, until the rate of angular momentum transfer ceases almost entirely by $t \simeq 1600$, at which point the distortion in the halo has become aligned with the bar.

As usual, the bar suffers a bending instability in the early stages (e.g., Raha et al. 1991). It is first detectable at $t \simeq 250$ and is over by $t \simeq 450$. In this model, the bar amplitude is not greatly affected by the buckling instability and the torque on the halo is only slightly reduced by this event.

The pattern speed of the bar also begins to decrease after its formation, as shown in Figure 2(b). Figure 2(c) shows that the angular momentum remaining in the inner part of the disc (where the bar resides) decreases in a similar fashion. The similar shapes of these two curves indicates that the bar has a positive moment of inertia which is approximately, though not exactly, constant, justifying Weinberg's original assumption.

It is interesting that the secular changes seem to end before the halo was brought to co-rotate with the bar. Late in the simulation, the mean angular rotation rate of the halo particles in the very centre is about half that of the bar, but the rotation of the outer halo is characterised more by a constant mean orbital speed rather than by uniform rotation. The alignment of the halo distortion with the bar appears to indicate that a large fraction of the halo particles are trapped into resonances with the bar.

As evolution appeared to have almost ceased, we stopped the calculation at $t = 2000$, which corresponds to 40 rotation periods at the initial rotation rate of the bar.

Fig. 2. (a) The time variations of the total angular momenta of the disc and halo, (b) of the bar pattern speed and (c) of the total angular momentum of particles in the inner and outer disc.

Fig. 3. The time variations of D_{L} (squares) and of a_{B} (triangles).

Our principal result is displayed in Figure 3, which shows estimates of the bar length and co-rotation radius at many times during the run. The distance, D_{L}, is that from the centre to the Lagrange point on the bar major axis, and is determined from the potential and pattern speed at each instant. The semi-major axis of the bar, a_{B}, is estimated to lie where the $m = 2$ coefficients of a Fourier expansion of the particle distribution depart from the constant phase and linear fall-off in amplitude characteristic of the outer bar region. This definition is consistent with those adopted by many observers.

The ratio $D_{\mathrm{L}}/a_{\mathrm{B}}$ increases steadily from a value of ~ 1.3 at $t = 200$, reaching ~ 3 by $t = 1200$. A large value of this ratio is quite unlike the values believed to pertain in real barred galaxies, as we review next.

3 Pattern Speeds of Bars in Galaxies

The only known technique to estimate the pattern speed directly from observations was proposed by Tremaine & Weinberg (1984b). It has been

successfully applied in just one case, the SB0 galaxy NGC 936: Merrifield & Kuijken (1995) considerably improved Kent's (1987) original measurement for this galaxy. It is to be hoped that this technique will soon be applied to more galaxies, though it is unlikely to be successful for later Hubble types.

The new spectroscopic and photometric data on NGC 936 yield an estimate of $\Omega_p \sin i = 3.1 \pm 0.75$ km s^{-1} arcsec^{-1}, which when combined with Kormendy's (1983) estimated inclination $i = 41^\circ$ and rotation curve, places co-rotation at a distance of 69 ± 15 arcsec from the centre of the galaxy. This is somewhat outside the visible bar which was estimated (Kent & Glaudell 1989) to end at about 50 arcsec from the centre. For this galaxy, therefore, $D_\mathrm{L}/a_\mathrm{B} = 1.4 \pm 0.3$.

All other techniques to estimate this ratio in galaxies are indirect. The best such evidence comes from the locations and shapes of dust lanes; the extensive survey of hydrodynamical bar flows by Athanassoula (1992) led her to conclude that $D_\mathrm{L}/a_\mathrm{B} = 1.1 \pm 0.1$ would best account for the dust lane morphology in galaxies. Her work was, however, restricted to rather artificial Ferrers bar models, which do not correspond well to the observed light distributions in bars. More recently Weiner (1996), P. Lindblad (this meeting) and others adopt mass models derived from the measured light distribution; preliminary results suggest that bars are indeed rotating rapidly, and that $D_\mathrm{L}/a_\mathrm{B} > 1.7$ seems to be firmly excluded.

There is evidence (Binney et al. 1991, Weiner & Sellwood 1996, Kalnajs this meeting) that the bar which is believed to reside in the Milky Way also has a high pattern speed, but the ratio $D_\mathrm{L}/a_\mathrm{B}$ is not yet firmly established.

Finally, it should be admitted that theoretical prejudice, which was originally the strongest "evidence" for fast bars (see Sellwood & Wilkinson 1993), has turned out to be almost worthless, since the bar in our simulation seems to survive quite happily with a low pattern speed!

While the above evidence could scarcely be described as overwhelming, it clearly favours values for $D_\mathrm{L}/a_\mathrm{B}$ that are quite inconsistent with those we measure from our simulation, at least after the first few bar rotations.

4 What Could be Wrong?

One's first reaction to such a puzzling result is to question whether it needs to be taken seriously. Of course it is reassuring that the simulation behaved as theory had already predicted and that similar results are also being obtained by Athanassoula (1996) using a quite different N-body method. But perhaps the model differs from real galaxies in respects which

cause it to severely overestimate the importance of dynamical friction, or that something has been omitted which would counteract the behaviour.

4.1 Friction Overestimated?

Chandrasekhar's formula (eq. 1), and Weinberg's analysis, indicates that the deceleration rate should be proportional to the bar mass and the halo density. Could either, or both, of these parameters be too large in our model?

Bar Strength It is widely believed that bars are massive features, at least in some galaxies. They are observed to give rise to strong non-circular motions in many well studied cases: good examples of strongly non-axisymmetric gas motions are seen in NGC 5383 (Sancisi, Allen & Sullivan 1979, Duval & Athanassoula 1983), NGC 1365 (Jörsäter & van Moorsel 1995) and NGC 4123 (Weiner 1996). A similar streaming pattern in the stellar motions in NGC 936 was observed by Kormendy (1983). (It should be noted that such streaming patterns are easily masked in galaxies where the bar lies close to one of the principal axes of the projected disc.) Such large non-circular motions seem to indicate a strongly non-axisymmetric potential, which in turn implies that the bar has a significant mass compared with the axisymmetric components in the central parts of these galaxies.

In order to be more quantitative, we have made a comparison between Kormendy's (1983) slit observations of the barred galaxy in NGC 936 with similar data taken from our model at $t = 250$ projected and inclined as in NGC 936. (This time was chosen because $D_{\rm L}/a_{\rm B}$ was close to the the value of 1.4 seen in NGC 936.) We estimate normalized $m = 2$ Fourier coefficients of both the radial and tangential velocities at fixed deprojected radii. Averaging over a range of radii near the end of the bar, we find we find the coefficients from our model are 2.8 and 1.7 times larger, for the azimuthal and radial components respectively, than the same values in NGC 936. We conclude that our bar is perhaps twice as strong as that in a typical barred galaxy, which may therefore have caused us to overestimate the spin down rate by a factor of two.

Halo Mass On the other hand, our halo is nowhere near massive or extensive enough to give rise to a flat rotation curve beyond the disc edge (Figure 1). Halo particles that never come close to it would clearly be unaffected by the bar, but one would expect the halo out to radii several times the bar semi-major axis ($a_{\rm B} \simeq 3$ for our bar) to be torqued

Fig. 4. The mean specific angular momenta of halo particles at $t = 2000$.

up by the bar. Figure 4 shows the mean specific angular momenta of halo particles at one instant late in the simulation, plotted as a function of radius, and indicates that particles far out in the halo have in fact gained disproportionately more angular momentum than those close in. Thus if we were to have run a simulation identical in most respects but having more mass in the outer halo to give a flat rotation curve, it seems likely that the bar would lose much more angular momentum. The under-massive halo of our model therefore gives too *little* dynamical friction.

Halo Core Radius Although we do not expect the friction force on a strong bar to behave precisely as equation (1) would predict, that formula does suggest that the braking rate should scale with the density and depend on the halo velocity dispersion. For an isothermal halo with a core, the velocity dispersion (assumed isotropic) is largely set by the circular velocity at large radii; there is therefore little freedom to juggle this parameter for realistic halos. On the other hand, a halo having a larger core radius will have a weaker effect, but only to the extent that friction arises from the inner halo.

At the cost of eliminating any effective bulge component, we could decrease the central density of our halo (see Figure 1). It cannot be decreased indefinitely, however, since the core radius of a realistic halo cannot be so large, relative to the disc scale, as to allow the rotation curve to decline significantly outside the disc. Thus observed asymptotically flat rotation curves require a minimum central halo density and a fixed velocity dispersion at large radii. Since we have already shown that halo mass at large radii takes up most of the angular momentum, we do not expect that a change to the central density, while keeping the halo mass fixed, will affect friction very much. Additional experiments to verify this expectation seem desirable.

Halo Rotation Halos are not expected to have large angular momenta (e.g., Barnes & Efstathiou 1987). We have, nevertheless, tested the possibility that halo rotation could reduce the bar spin down rate by running two further simulations, identical in all respects except that in one case some fraction of the retrograde halo particles had their angular momenta flipped to give a total halo angular momentum about half the maximum possible. We found the bar pattern speed to drop by about the same amount in both and therefore conclude that dynamical friction is not significantly decreased by giving the halo even a large positive angular momentum.

4.2 Effects Omitted?

Secondary Bar Growth Sellwood (1981) found that when a small bar formed within an extensive disc, it could grow in length due to trapping of additional stars into the bar as some angular momentum is removed by spirals in the outer disc. In his most extreme case, the bar's half-length approximately doubled from its initial value. Since the disc in our simulation was initially truncated at $R = 4$ and the bar which formed had a semi-major axis of fully half the distance to the initial edge, the scope for significant secondary bar growth is severely limited in our present model.

Could such substantial bar growth account for the small $D_\mathrm{L}/a_\mathrm{B}$ ratios of real galaxies? We do not think it likely for two main reasons: first, the bar would have to grow continually which requires incessant spiral activity in the outer disc. This is manifestly not happening now in the SB0 galaxy NGC 936; the bar in this galaxy is likely to have formed some time ago and with little sign of spiral activity in the outer disc, it cannot have grown much recently. Yet a low value of $D_\mathrm{L}/a_\mathrm{B}$ seems well established in this

particular galaxy. The other reason is that secondary bar growth makes the bar longer and stronger, which would increase dynamical friction and therefore have a less than totally beneficial effect. Preliminary results from a further experiment seem to confirm that more extensive discs do not in fact lead to significantly smaller $D_{\rm L}/a_{\rm B}$ ratios.

Bar Spin-up Our simulations are purely stellar and ignore the effects of gas. It is well known that the offset shocks on the leading side of the bar cause the gas to lose angular momentum. That angular momentum is, of course, given up to the bar. However, the amount of angular momentum is quite insignificant, since the gas mass is already small and the lever arm associated with it is short.

Radial inflows of gas are of slightly greater importance, however. Increases in the central mass concentration affect the potential in which the bar resides, and one consequence is an increase in the bar pattern speed (see also Kalnajs, this meeting). In §6.1 we give an example in which a substantial mass influx causes the bar pattern speed to rise by some 25%. This is helpful, but on its own, utterly inadequate to reconcile our simulation with observations.

5 Assessment

Thus far we have demonstrated that Weinberg's theoretical prediction of strong dynamical friction is at least qualitatively confirmed and that the bar is braked rapidly to an angular rate which is quite inconsistent with observed $D_{\rm L}/a_{\rm B}$ ratios. We here list the possible solutions to this discrepancy between theory and observation that have occurred to us or been suggested by others.

1. Bars have low pattern speeds
2. Bars are weak
3. Bars grow in length as they slow down
4. Bars are spun up – e.g., by gas inflow
5. Bars have enormous effective moments of inertia
6. The halo co-rotates with the bar
7. Many halo particles are locked into resonance with the bar
8. Bars do not last long
9. Halos are not very massive

The entire problem hinges on alternative 1 being excluded. The evidence for fast bars (§3) is not as strong as we would wish, and we are

uncomfortable that rather too many of our arguments rest on the assumption that the early-type SB0 galaxy NGC 936 is typical. The evidence for massive halos is strongest for unbarred, late-type spiral galaxies (see §7). More data confirming both a high pattern speed and a massive halo in several barred galaxies would be most welcome.

We have disposed of possibility 2 and argued that 3 & 4 are minor effects that could do little towards removing the discrepancy. The moment of inertia of the bar in our simulation, at least, is not large enough to prevent dynamical friction from slowing it; it seems unlikely that the structure of real bars is sufficiently different to change this conclusion. Alternative 6 also does not deserve lengthy consideration – the angular momentum of the halo would have to be inconceivably large.

Alternative 7 is somewhat more interesting. Dynamical friction in our simulation all but ceases while the bar rotated significantly, which seems to indicate that many halo particles have become trapped in resonances. This phenomenon deserves further investigation, but it is clear that it cannot provide a solution to our puzzle since friction ceases only after the bar pattern speed has dropped by a factor of five.

The remaining two alternatives are much more radical, but have to be contemplated since no other solutions seem tenable.

6 Transient Bars?

The possibility that bars could disappear before dynamical friction had sufficient time to slow them down was first suggested by Hernquist & Weinberg (1992). In order not to violate the bound of $D_{\rm L}/a_{\rm B} < 1.7$ suggested by observation, most bars would have to be destroyed quite quickly – within 10 rotations, judging from our simulation. Thus, to maintain the observed substantial fraction of galaxies containing strong bars (e.g., Sellwood & Wilkinson 1993), this idea requires bars to form and dissolve more than once over the lifetime of a galaxy. A second attraction of such a radical idea is that the fraction of galaxies containing strong bars, for which there is still no convincing explanation, represents a 30% duty cycle in the barred phase. Regenerating a bar in a disc where one has previously been destroyed presents a formidable problem, however.

6.1 Bar Destruction

Many authors have noted that bars are robust, long-lived systems that are not easily destroyed. Our simulation provides yet another example;

Fig. 5. The projected distribution of disc particles at $t = 2000$ showing that a strong, butterfly-shaped bar survives. We have not included any bulge/halo particles.

despite having lost some 2/3 of its angular momentum and having reduced its pattern speed by a factor of 5, it remains a strong bar, as shown in Figure 5. Thus our simulation excludes the possibility, left open by Weinberg's analysis, that bars simply would not survive such fierce braking.

There are just two known ways to destroy bars: one obvious way is to hit the bar with a companion, the other is to have a build-up of mass at the bar centre. As Athanassoula (this meeting) presents a major study of bar-satellite interactions, we do not discuss them here.

The effects of central mass concentrations have been explored exten-

Fig. 6. The time dependence of the bar pattern speed in the 3-D model of Norman et al. (1996). The non-disc components of this model were rigid but the bar still slows through interactions with the outer disc until $t = 100$. Mass influx was mimicked by shrinking a rigid Plummer sphere component having 5% of the disc + bulge mass. The scale radius of this component decreased by a factor of 40 over the period $100 < t < 150$, which caused the pattern speed to rise before the bar dissolved at $t \simeq 130$.

sively (Hasan & Norman 1990, Hasan, Pfenniger & Norman 1993, Wada & Habe 1992, Friedli & Benz 1993, 1995, Heller & Shlosman 1994 and Norman, Sellwood & Hasan 1996). The idea here is that the gas driven towards the centre by the bar itself changes the gravitational potential within the bar to a sufficient extent that the main orbit family (Contopoulos's family x_1) becomes chaotic, and the regular part of phase space switches to the perpendicular x_2 family. A self-consistent bar can no longer survive once this happens, and the bar becomes a spheroidal bulge (Norman et al. 1996). The precise central mass and degree of concentration needed to achieve this has yet to be firmly tied down, but a few percent of the total mass of the disc seems ample.

Returning for a moment to the point made in §4.2, we present Figure 6 to illustrate that the simple process of forming a central mass concentration increases the bar pattern speed. This result is taken from the 3-D simulation by Norman et al. in which the mass build-up was mimicked by simply contracting a rigid spherical mass component containing 5% of

the disc and bulge mass. The increase in pattern speed (by some 25% in this case) therefore cannot have been caused by external torques and is simply a result of the changing internal structure of the bar.

6.2 Difficulties with Regenerating Bars

Destruction of a bar, either by the above mechanism or by interaction with a satellite, would leave the disc in a dynamically very hot state. The processes of both its formation and destruction would disturb the disc stars into quite markedly eccentric orbits, making the disc quite unresponsive to the kind of large-scale collective instability needed to reform a bar. Since only gas can cool, the galaxy would require a long recuperation period in which a large supply of fresh gas led to the formation of a substantial fraction of new stars on nearly circular orbits before the disc could become receptive to a new global instability. While the demands here seem excessive, this process could conceivably occur in the gas-rich late Hubble types; the theory would therefore appear to predict an increasing bar frequency along the Hubble sequence that is not observed (Sellwood & Wilkinson 1993).

Furthermore, if most bars are destroyed by central mass concentrations, the galaxy will be made more stable. A high central density is precisely what is required to stabilize a massive disc (Toomre 1981 and this meeting). This is not a watertight argument, since it is not clear that the galaxy would be absolutely stable no matter how cool the disc, and bars could also be triggered by interactions (e.g, Noguchi 1987), but it adds considerably to the difficulties faced by the recurrent bar idea.

Since both methods of bar dissolution make a bulge, bars in bulgeless galaxies must therefore be their first which, unless they are rotating slowly, would be required in this picture to be young. It is perhaps interesting that most bulgeless barred galaxies are low luminosity galaxies. Unfortunately, it is unclear what observational data on such galaxies would be required to test the prediction of dynamical youthfulness.

The idea of transient bars therefore faces extreme challenges. They would be reduced somewhat if one could argue that the first bar in a galaxy is braked by the halo, which is then sufficiently spun-up as to exert much weaker friction on a second bar. Some such wildly speculative idea is required if the regenerated bars alternative is to remain viable.

7 Low Mass Halos?

The final possible alternative is that galaxies with fast bars lack massive halos. The best evidence for massive halos comes from the extended, flat

HI rotation curves in late-type, unbarred spiral galaxies (e.g., van Albada & Sancisi 1986). Occam's razor, together with current ideas of galaxy formation, suggest that all galaxies should have flat outer rotation curves, but the supporting observational data is still sketchy. Bosma (1992 and this meeting) concludes that barred galaxies generally do have extensive flat rotation curves. An exception for NGC 1365 is claimed by Jörsäter (this meeting, Jörsäter & van Moorsel 1995), but deprojection of the complex kinematic map of this strongly barred, asymmetric, and probably also warped galaxy is exceedingly difficult. The evidence for massive halos in early-type galaxies is also weak because they generally lack the gas disc which makes such a useful tracer of the potential in late-type systems. van Driel and collaborators have attempted to address this issue by mapping the gas in those rare S0 galaxies that are relatively gas rich, finding some evidence for flat rotation curve at large radii in the case of NGC 4203 (van Driel et al. 1988).

We therefore think it likely that the circular velocity stays high at large radii in all galaxies, including those with fast bars. If this does not indicate a massive halo, then some alternative explanation for the phenomenon would need to be invoked (e.g, Milgrom & Bekenstein 1987).

8 Conclusions

The pattern speed problem presented by dynamical friction between a bar and bulge/halo is becoming rather insistent. Most possible solutions seem unattractive, some are excluded and others need to be stretched excessively. It is becoming increasingly difficult to find a tenable conventional explanation.

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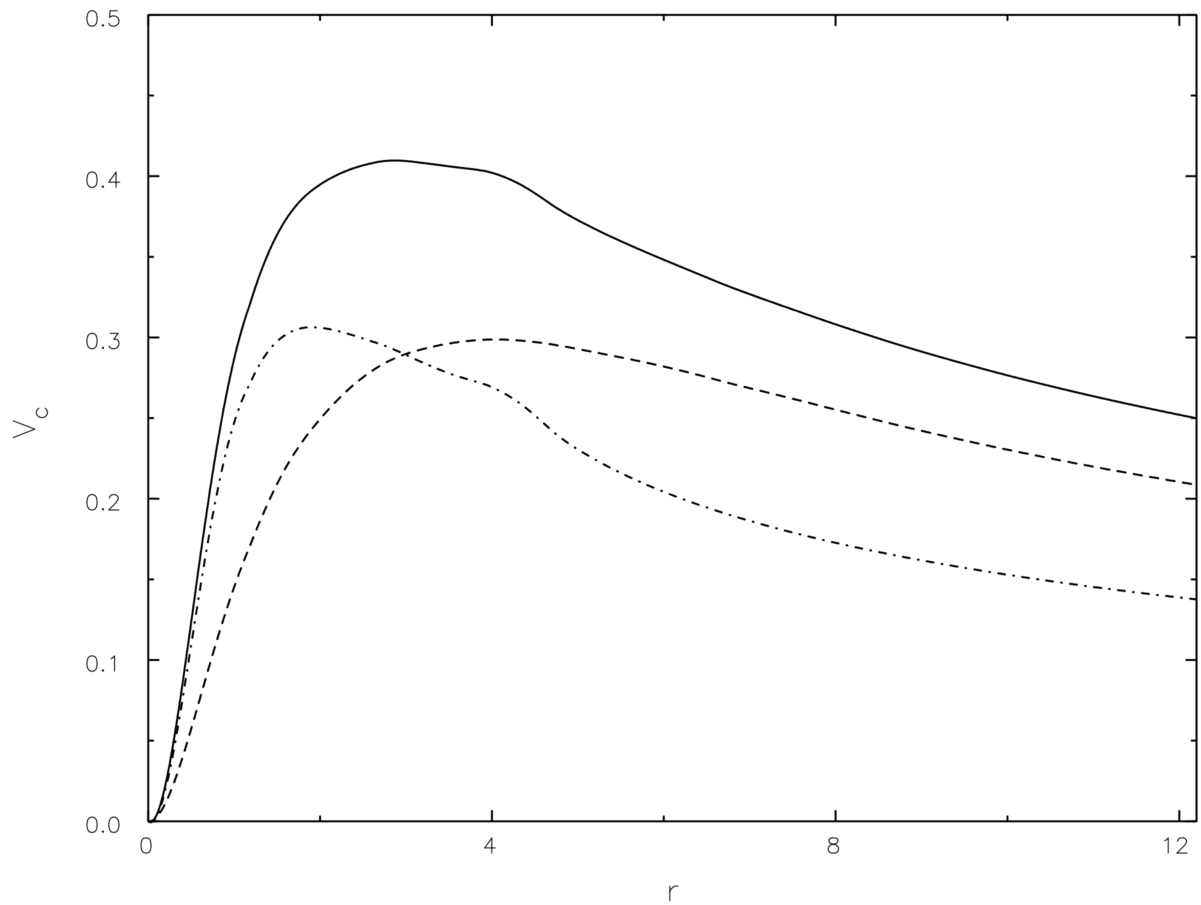

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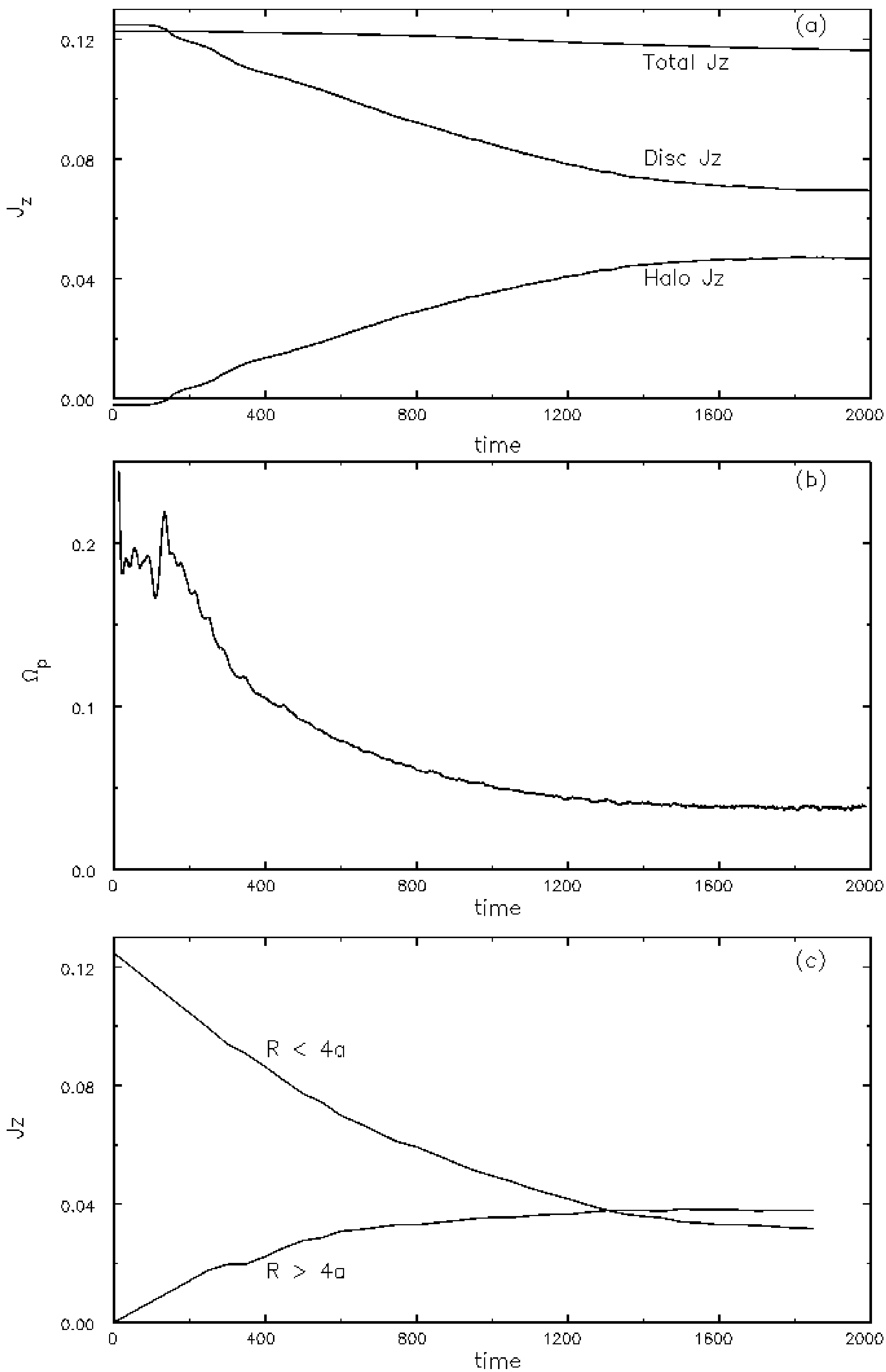
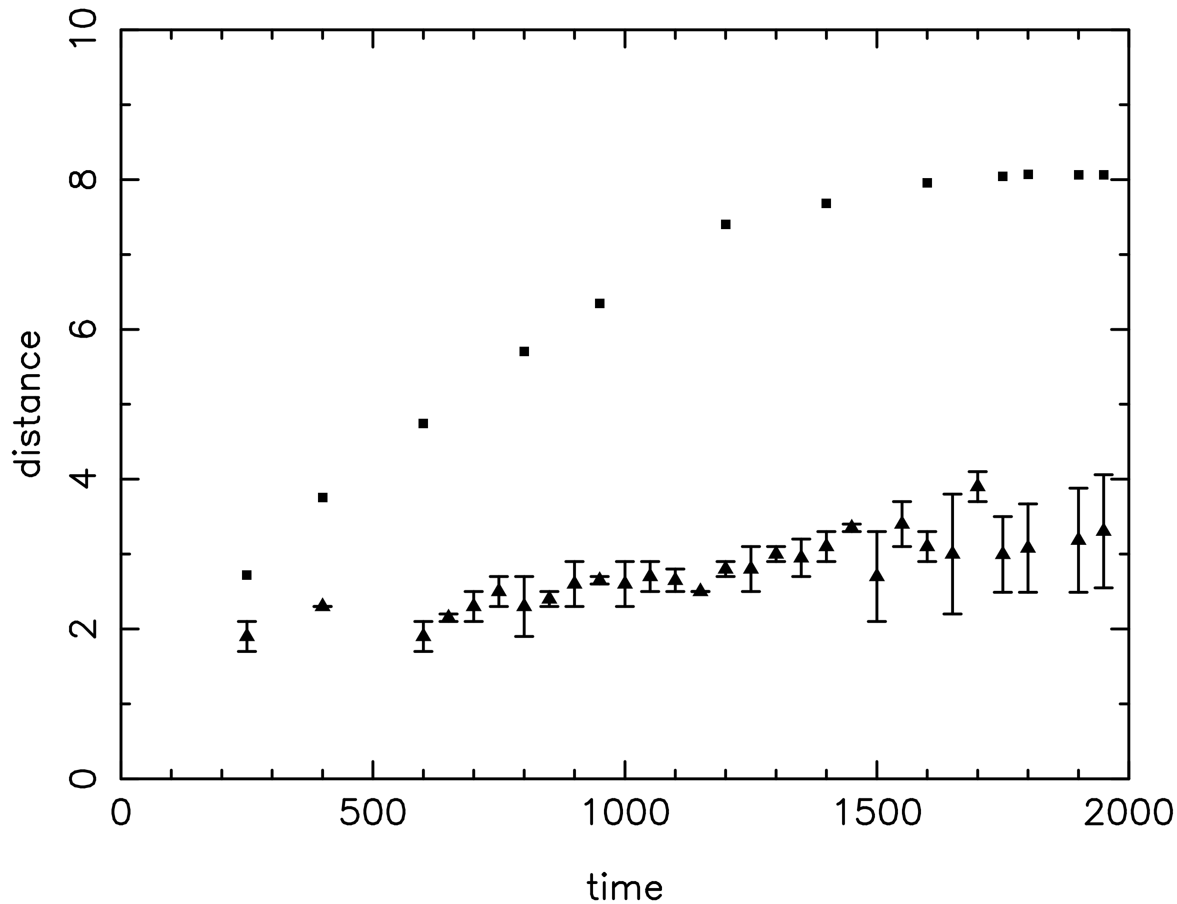
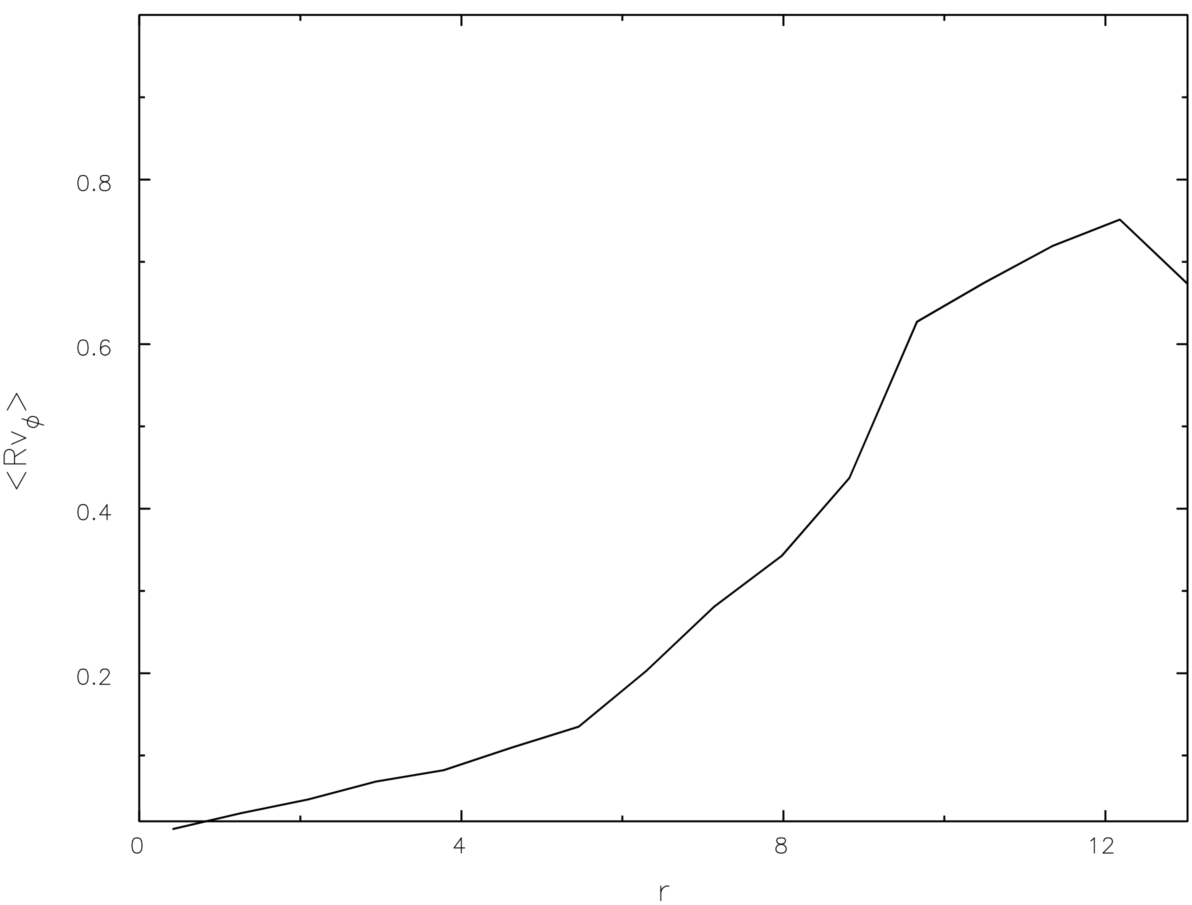
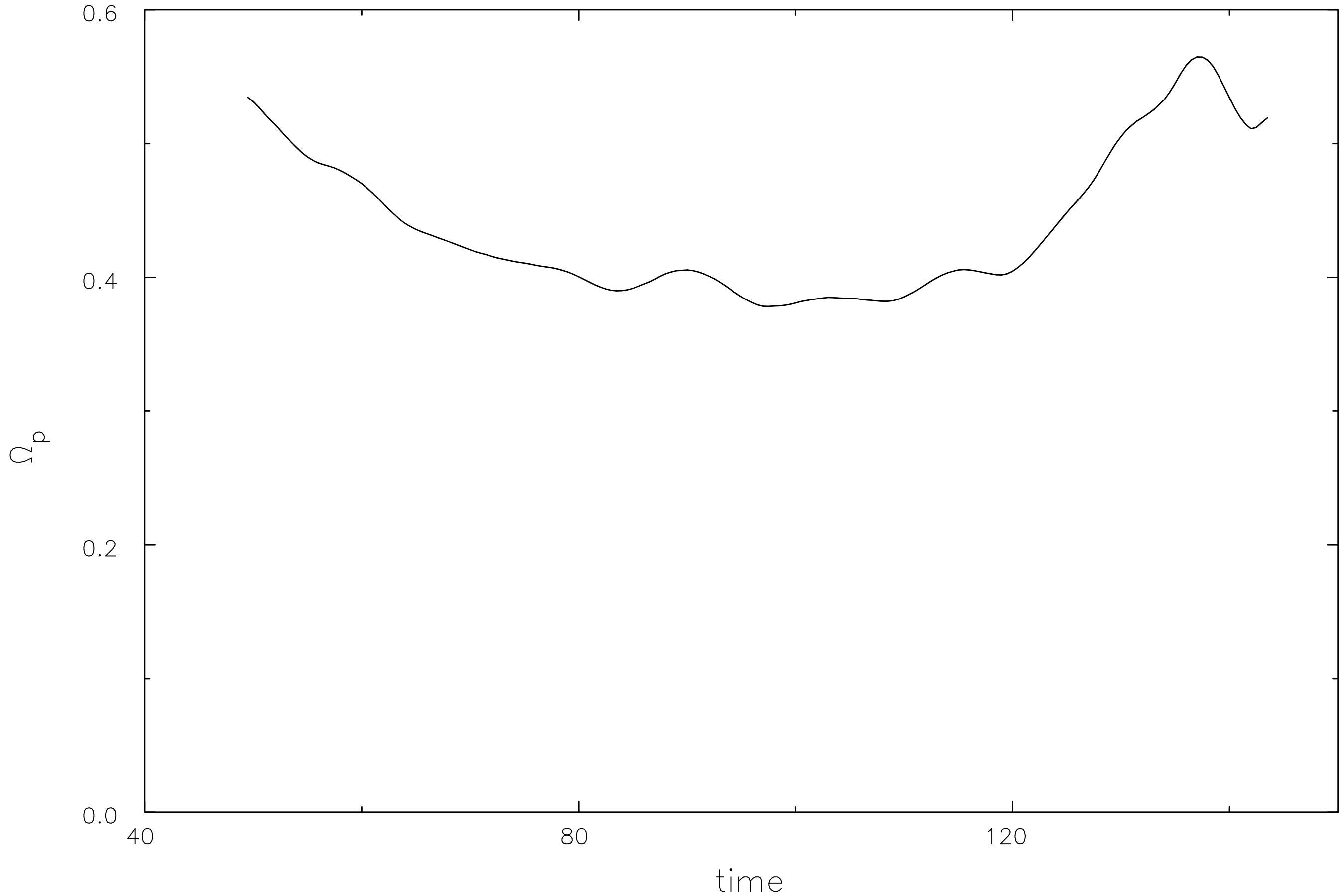
12
r


(a)
Total Jz
Disc Jz
Halo Jz
J_z
0.00
0.04
0.08
0.12
0
400
800
1200
1600
2000
time
(b)
Ω_p
0.0
0.1
0.2
0
400
800
1200
1600
2000
time
(c)
R < 4a
R > 4a
Jz
0.00
0.04
0.08
0.12
0
400
800
1200
1600
2000
time



distance
10
8
6
4
2
0
0
500
1000
1500
2000
time


0.8
0.6
0.4
0.2
0
4
8
12
<Rv$_\phi$>
r



0.6
0.4
0.2
0.0
Ω_p
40
80
120
time